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# ANALYTIC FORM OF THE ONE-LOOP VERTEX AND OF THE TWO-LOOP FERMION PROPAGATOR IN 3-DIMENSIONAL MASSLESS QED

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ABSTRACT

We evaluate the analytic expression for the one-loop fermion-boson vertex in massless QED3 in an arbitrary covariant gauge. The result is written in terms of elementary functions of its momenta. The vertex is decomposed into a longitudinal part, that is fully responsible for ensuring the Ward and Ward-Takahashi identities are satisfied, and a transverse part. Following Ball and Chiu and Kızılersü *et. al.*, the transverse part is written in its most general form as a function of 4 independent vectors. We calculate the coefficients of each of these vectors. We also check the transversality condition to two loops and evaluate the fermion propagator to the same order. We compare our results with a conjecture of the non-perturbative vertex by Tjiang and Burden.

# 1 Introduction

QED in 3-dimensions is a useful laboratory to study Schwinger-Dyson equations. As compared to its 4-dimensional counterpart, it is relatively simpler because of the lack of ultraviolet divergences. Moreover, in the quenched approximation, it exhibits confinement which makes it more attractive for non-perturbative studies. The non-perturbative study of gauge theories through the use of Schwinger-Dyson equations requires the knowledge of the non-perturbative form of the fundamental fermion-boson interaction. The most commonly used approximation is the bare vertex. However, among other drawbacks, it fails to respect a key property of a gauge field theory, namely the gauge invariance of physical observables. An obvious reason is that the bare vertex fails to respect the Ward Takahashi Identity (WTI). Ball and Chiu [1] have proposed an ansatz for what is conventionally called the longitudinal part of the vertex which alone satisfies WTI. Another transverse part remains undetermined. The QED3 (quenched and unquenched) has been studied for dynamical mass generation for fermions in the bare vertex approximation as well as using an *ansatz* for the full vertex which is a simple modification of that proposed by Ball and Chiu [2, 3]. More recently, another full vertex *ansatz* has been used to study fermion and photon propagators simultaneously [4], including an explicit transverse piece.

The only truncation of the complete set of Schwinger-Dyson equations known so far that incorporates the gauge invariance of a gauge theory at each level of approximation is perturbation theory. Therefore, it is natural to assume that physically meaningful solutions of the Schwinger-Dyson equations must agree with perturbative results in the weak coupling regime. It requires, e.g., that every non-perturbative *ansatz* chosen for the full vertex must reduce to its perturbative counterpart when the interactions are weak. Whereas in QED4 this realization has been of enormous help to construct physically acceptable form of the vertex [5, 6, 7], need exists to exploit perturbation theory in exploring the non-perturbative form of the vertex in QED3. Following [6], we perform an analogous calculation in QED3. We evaluate one-loop vertex in perturbation theory for massless fermions. Unlike QED4, all the loop integrals involved are perfectly well-behaved in ultraviolet regime and hence there is no need to renormalize them.

The fermion propagator, $S_F$, of momentum $p$ involves only one function of $p^2$ for massless fermions. We call it $F(p^2)$.

$$iS_F(p) = i\frac{F(p^2)}{\not p} \qquad . \tag{1}$$

The Ward-Takahashi identity relates the 3-point Greens function to the fermion propagator. The work of Ball and Chiu [1] tells us how to express the non-perturbative structure of the longitudinal part of the vertex in terms of $F(p^2)$. The knowledge of the fermion function $F(p^2)$ helps us evaluate the longitudinal component of the vertex. The transverse vertex is obtained by subtracting the longitudinal one from the full vertex. In its most general form, the full vertex can be written in terms of 12 basis tensors. The Ball-Chiu construction consumes 4 of these to write the longitudinal vertex and 8 are left to express the transverse part. For massless fermions, only 4 of the 8 coefficients are non-vanishing. The vertex should be free of any kinematic singularities. Ball and Chiu choose the basis in such a way that the coefficient of each of the basis is independently free of kinematic singularities in the Feynman gauge. This basis was later modified to exhibit the same quality in an arbitrary covariant gauge by Kızılersü *et. al.* [6]. There is no a priori reason for the coefficients to be free of kinematic singularities in QED3 as well with the same choice of basis. However, we find that the same set of basis vectors serve perfectly well for QED3 as well. We present the final expression for all the coefficients in terms of basic functions of the momenta involved. This result should serve as a guide in hunting for the non-perturbative form of the transverse vertex as every such construction should reduce to it in the weak coupling regime. We also check the transversality condition to two loops and find that to this order, it is not realized in perturbation theory. We evaluate $F(p^2)$ to $\mathcal{O}(\alpha^2)$ analytically and compare our findings with a recent conjecture of the vertex proposed by Tjiang and Burden.

# 2 The Full Vertex

## 2.1 The Non-perturbative Vertex

The full vertex, Fig. 1, $\Gamma^\mu(k,p)$ can be expressed in terms of 12 spin amplitudes formed from the vectors $\gamma^\mu, k^\mu, p^\mu$ and the scalars $1, \not{k}, \not{p}$ and $\not{k}\,\not{p}$. Thus we can write

$$\Gamma^\mu = \sum_{i=1}^{12} P^i V_i^\mu \qquad , \tag{2}$$

where we choose the $V_i^\mu$ as follows

$$\begin{aligned} V_1^\mu &= k^\mu \not{k} \ , \quad V_2^\mu = p^\mu \not{p} \ , \quad V_3^\mu = k^\mu \not{p}, \quad V_4^\mu = p^\mu \not{k} \\ V_5^\mu &= \gamma^\mu \not{k}\not{p}, \quad V_6^\mu = \gamma^\mu \ , \quad V_7^\mu = k^\mu \ , \quad V_8^\mu = p^\mu \\ V_9^\mu &= p^\mu \not{k}\not{p}, \quad V_{10}^\mu = k^\mu \not{k}\not{p}, \quad V_{11}^\mu = \gamma^\mu \not{k}, \quad V_{12}^\mu = \gamma^\mu \not{p} \qquad . \end{aligned} \tag{3}$$

The full vertex satisfies the Ward-Takahashi identity

$$q_\mu \Gamma^\mu(k,p) = S_F^{-1}(k) - S_F^{-1}(p), \tag{4}$$

where $q = k - p$, and the Ward identity

$$\Gamma^\mu(p,p) = \frac{\partial}{\partial p^\mu} S_F^{-1}(p) \tag{5}$$

as the non-singular $k \to p$ limit of Eq. (4). We follow Ball and Chiu and define the longitudinal component of the vertex in terms of the fermion propagator as

$$\Gamma_L^\mu \;=\; \frac{\gamma^\mu}{2}\left(\frac{1}{F(k^2)} + \frac{1}{F(p^2)}\right) \;+\; \frac{1}{2}\,\frac{(\not k + \not p)(k+p)^\mu}{(k^2-p^2)}\left(\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right) . \tag{6}$$

$\Gamma_L^\mu$ alone then satisfies the Ward-Takahashi identity, Eq. (4), and being free of kinematic singularities the Ward identity, Eq. (5), too. The full vertex can then be written as

$$\Gamma^\mu(k,p) = \Gamma_L^\mu(k,p) + \Gamma_T^\mu(k,p) \qquad , \tag{7}$$

where the transverse part satisfies

$$q_\mu \Gamma_T^\mu(k,p) = 0 \quad \text{ and } \quad \Gamma_T^\mu(p,p) = 0 \qquad . \tag{8}$$

The Ward-Takahashi identity fixes 4 coefficients of the 12 spin amplitudes in terms of the fermion functions — the 3 combinations explicitly given in Eq. (6), while the coefficient of $\sigma_{\mu\nu} k^\mu p^\nu$ must be zero [1]. The transverse component $\Gamma_T^\mu(k,p)$ thus involves 8 vectors, out of which the following 4 are sufficient to describe the transverse vertex for the case of massless fermions :

$$\Gamma_T^\mu(k,p) = \sum_{i=2,3,6,8} \tau_i(k^2,p^2,q^2)\, T_i^\mu(k,p) \qquad , \tag{9}$$

where

$$\begin{aligned}
T_2^\mu &= \left[p^\mu (k\cdot q) - k^\mu (p \cdot q)\right](\not k + \not p) \\
T_3^\mu &= q^2 \gamma^\mu - q^\mu \not q \\
T_6^\mu &= \gamma^\mu (p^2 - k^2) + (p+k)^\mu \not q \\
T_8^\mu &= -\gamma^\mu k^\nu p^\lambda \sigma_{\nu\lambda} + k^\mu \not p - p^\mu \not k \\
\text{with} \qquad \sigma_{\mu\nu} &= \frac{1}{2}[\gamma_\mu, \gamma_\nu] \qquad .
\end{aligned} \tag{10}$$

The coefficients $\tau_i$ are Lorentz scalar functions of $k$ and $p$, i.e., functions of $k^2, p^2, q^2$.

## 2.2 The one loop calculation

The vertex of Fig. 1 can be expressed as

$$\Gamma^\mu(k,p) = \gamma^\mu + \Lambda^\mu(k,p). \tag{11}$$

Using the Feynman rules, $\Lambda^\mu$ to $O(\alpha)$ is simply given by:

$$-\,ie\Lambda^\mu = \int_M \frac{d^3w}{(2\,\pi)^3}(-ie\gamma^\alpha)iS_F^0(p-w)(-ie\gamma^\mu)iS_F^0(k-w)(-ie\gamma^\beta)i\Delta^0_{\alpha\beta}(w)\ , \tag{12}$$

where $M$ denotes the loop integral is to be performed in Minkowski space. The bare quantities are

$$\begin{aligned} -ie\Gamma^0_\mu &= -ie\gamma_\mu \\ iS_F^0(p) &= i\not{p}/p^2 \\ i\Delta^0_{\mu\nu}(p) &= -i\left[p^2 g_{\mu\nu} + (\xi-1)p_\mu p_\nu\right]/p^4 \quad , \end{aligned}$$

where $e$ is the usual QED coupling and the parameter $\xi$ specifies the covariant gauge. Substituting these values in Eq. (12), we have with $\alpha \equiv e^2/4\pi$:

$$\Lambda^\mu = -\frac{i\,\alpha}{2\,\pi^2}\int_M d^3w \left\{\frac{A^\mu}{w^2\,(p-w)^2\,(k-w)^2} + (\xi-1)\frac{B^\mu}{w^4\,(p-w)^2\,(k-w)^2}\right\} \tag{13}$$

on separating the $g_{\alpha\beta}$ and $w_\alpha w_\beta$ parts of the photon propagator. In the above equation,

$$A^\mu = \gamma^\alpha\ (\not{p} - \not{w})\ \gamma^\mu\ (\not{k} - \not{w})\ \gamma_\alpha \quad , \tag{14}$$

$$B^\mu = \not{w}\ (\not{p} - \not{w})\ \gamma^\mu\ (\not{k} - \not{w})\ \not{w} \quad . \tag{15}$$

What makes the present calculation different from the one in 4-dimensions is that due to the reduction of powers of $w$ in the numerator, none of the integrals is ultraviolet divergent. To proceed we introduce the following seven basic integrals over the loop momentum $d^3w$ : $J^{(0)}, J^{(1)}_\mu, J^{(2)}_{\mu\nu}, I_0, I^{(1)}_\mu, I^{(2)}_{\mu\nu}$ and $K^{(0)}$.

$$J^{(0)} = \int_M d^3w\,\frac{1}{w^2\,(p-w)^2\,(k-w)^2} \tag{16}$$

$$J^{(1)}_\mu = \int_M d^3w\,\frac{w_\mu}{w^2\,(p-w)^2\,(k-w)^2} \tag{17}$$

$$J^{(2)}_{\mu\nu} = \int_M d^3w\,\frac{w_\mu w_\nu}{w^2\,(p-w)^2\,(k-w)^2} \tag{18}$$

$$I^{(0)} = \int_M d^3w\,\frac{1}{w^4\,(p-w)^2\,(k-w)^2} \tag{19}$$

$$I^{(1)}_{\mu} \;=\; \int_M d^3w \, \frac{w_\mu}{w^4\,(p-w)^2\,(k-w)^2} \tag{20}$$

$$I^{(2)}_{\mu\nu} \;=\; \int_M d^3w \, \frac{w_\mu w_\nu}{w^4\,(p-w)^2\,(k-w)^2} \tag{21}$$

$$K^{(0)} \;=\; \int_M d^3w \, \frac{1}{(p-w)^2\,(k-w)^2} \quad . \tag{22}$$

$\Lambda^\mu$ of Eq. (13) can then be re-expressed in terms of five of these as:

$$\Lambda^\mu \;=\; -\frac{i\,\alpha}{2\,\pi^2}\Big\{\gamma^\alpha\, \not{p}\, \gamma^\mu\, \not{k}\, \gamma_\alpha J^{(0)} - \gamma^\alpha\,(\not{p}\,\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu\,\not{k})\,\gamma_\alpha J^{(\mathbf{1})}_{\nu} + \gamma^\alpha\gamma^\nu\gamma^\mu\gamma^\lambda\gamma_\alpha J^{(\mathbf{2})}_{\nu\lambda}$$
$$+(\xi-1)\Big[\,(-\gamma^\nu\,\not{p}\,\gamma^\mu - \,\gamma^\mu\,\not{k}\,\gamma^\nu)\,J^{(\mathbf{1})}_{\nu} + \gamma^\mu K^{(0)} + \gamma^\nu\,\not{p}\,\gamma^\mu\,\not{k}\,\gamma^\lambda I^{(\mathbf{2})}_{\nu\lambda}\Big]\Big\}. \tag{23}$$

As the next step, we compute the basic integrals of Eqs. (16-22), [8, 9, 10] each of which is a function of $k$ and $p$.

## $J^{(1)}_{\mu}$ and $J^{(2)}_{\mu\nu}$ calculated:

Following [1, 6], we expand the Lorentz vector $J^{(1)}_{\mu}$ in its most general form in terms of the 4-momenta $k_\mu$ and $p_\mu$:

$$J^{(1)}_{\mu} = \frac{i\pi^3}{2}\,[k_\mu J_A(k,p) + p_\mu J_B(k,p)] \tag{24}$$

where $J_A$, $J_B$ must be scalar functions of $k$ and $p$. The factor of $i\pi^3/2$ is taken out purely for later convenience. One can see from the integral form of $J^{(1)}_{\mu}$ that $J_B(k,p) = J_A(p,k)$. Inverting the above equation,

$$J_A(k,p) = \frac{1}{i\pi^3\Delta^2}\left[2\,k\cdot p\,p^\mu J^{(1)}_{\mu} - 2\,p^2 k^\mu J^{(1)}_{\mu}\right] \tag{25}$$

with a similar expression for $J_B$, where $\Delta^2 = (k\cdot p)^2 - k^2p^2$. Moreover, using the identity

$$2p\cdot w \;=\; p^2 + w^2 - (p-w)^2 \tag{26}$$

for any 4-momentum $p$, we can write

$$k^\mu J^{(1)}_{\mu} \;=\; \frac{k^2}{2}\,J^{(0)} + \frac{1}{2}\,K^{(0)} - \frac{1}{2}\int \frac{d^3w}{w^2(p-w)^2} \tag{27}$$

and a similar expression for $p^\mu J^{(1)}_{\mu}$. Evaluating the scalar integrals involved using dimensional regularization and substituting the result in Eq. (25), we arrive at:

$$J_A(k,p) \;=\; \frac{1}{\Delta^2}\left\{\frac{p^2 k\cdot q}{\sqrt{-k^2p^2q^2}} \;+\; \frac{p\cdot q}{\sqrt{-q^2}} \;-\; \frac{k\cdot p}{\sqrt{-k^2}} \;+\; \frac{p^2}{\sqrt{-p^2}}\right\} \tag{28}$$

$$J_B(k,p) \;=\; J_A(p,k) \tag{29}$$

where we have made use of Eqs. (1,5) of the Appendix. In an analogous fashion, the tensor integral $J^{(2)}_{\mu\nu}$ of Eq. (18) can be expressed in terms of scalar integrals $K_0$,$J_C, J_D$ and $J_E$ by

$$\begin{aligned} J^{(2)}_{\mu\nu} &= \frac{i\pi^3}{2}\Bigg\{\frac{g_{\mu\nu}}{3}K_0 + \left(k_\mu k_\nu - g_{\mu\nu}\frac{k^2}{3}\right) J_C \\ &+ \left(p_\mu k_\nu + k_\mu p_\nu - g_{\mu\nu}\frac{2(k\cdot p)}{3}\right) J_D + \left(p_\mu p_\nu - g_{\mu\nu}\frac{p^2}{3}\right) J_E\Bigg\} \quad , \end{aligned} \tag{30}$$

where

$$J_C(k,p) = \frac{1}{2\Delta^2}\Bigg\{p^2(k\cdot p - 2k^2)\ J_A - p^4\ J_B + \frac{k\cdot p + p^2}{\sqrt{-q^2}} - \frac{k\cdot p}{\sqrt{-k^2}}\Bigg\}, \tag{31}$$

$$\begin{aligned} J_D(k,p) &= \frac{1}{4\Delta^2}\Bigg\{k^2(3k\cdot p - p^2)\ J_A + p^2(3k\cdot p - k^2)J_B - \frac{(k+p)^2}{\sqrt{-q^2}} \\ &\qquad + \frac{k^2}{\sqrt{-k^2}} + \frac{p^2}{\sqrt{-p^2}}\Bigg\}, \\ J_E(k,p) &= J_C(p,k)\ , \end{aligned} \tag{32}$$

which involve the previously found $J_A$ and $J_B$ of Eqs. (28,29).

**$I^{(1)}_\mu$ and $I^{(2)}_{\mu\nu}$ calculated:**

In a way analogous to the computation of $J^{(1)}_\mu$ and $J^{(2)}_{\mu\nu}$ the ultraviolet finite integrals $I^{(1)}_\mu$ and $I^{(2)}_{\mu\nu}$ [10] of Eqs. (20,21) can be re-expressed in terms of scalar integrals, $I_A, I_B, I_C, I_D, I_E$, that in turn involve the same functions we have already computed. Thus

$$I^{(1)}_\mu = \frac{i\pi^3}{2}\left[k_\mu I_A(k,p) + p_\mu I_B(k,p)\right] \quad , \tag{33}$$

where

$$I_A(k,p) = \frac{-1}{k^2}\frac{1}{\sqrt{-k^2p^2q^2}} \qquad \text{and} \qquad I_B(k,p) = I_A(p,k) \quad . \tag{34}$$

$I^{(2)}_{\mu\nu}$ can be expressed as

$$\begin{aligned} I^{(2)}_{\mu\nu} &= \frac{i\pi^3}{2}\Bigg\{\frac{g_{\mu\nu}}{3}J_0 + \left(k_\mu k_\nu - g_{\mu\nu}\frac{k^2}{3}\right) I_C \\ &\quad + \left(p_\mu k_\nu + k_\mu p_\nu - g_{\mu\nu}\frac{2(k\cdot p)}{3}\right) I_D + \left(p_\mu p_\nu - g_{\mu\nu}\frac{p^2}{3}\right) I_E\Bigg\}, \end{aligned} \tag{35}$$

where

$$I_C(k,p) \;=\; \frac{1}{2\Delta^2}\Bigg\{p^2(k\cdot p-2k^2)\;I_A - p^4\;I_B\;+(k\cdot p-2p^2)\;J_A - p^2\;J_B$$

$$-\;\frac{1}{\sqrt{-k^2}}\,\frac{k\cdot p}{k^2}\;-\;\frac{4p^2}{\sqrt{-k^2p^2q^2}}\Bigg\}\quad , \tag{36}$$

$$I_D(k,p) \;=\; \frac{1}{4\Delta^2}\Bigg\{k^2(3k\cdot p-p^2)\;I_A + p^2(3k\cdot p-k^2)\;I_B\;+(3k\cdot p-k^2)\;J_A$$

$$+\,(3k\cdot p-p^2)\;J_B\;+\;\frac{1}{\sqrt{-k^2}}\;+\;\frac{1}{\sqrt{-p^2}}\;+\;\frac{8k\cdot p}{\sqrt{-k^2p^2q^2}}\Bigg\}\quad , \tag{37}$$

$$I_E(k,p) \;=\; I_C(p,k)\quad . \tag{38}$$

**$\Lambda^\mu$ collected:**

$\Lambda^\mu$ can now be written completely in terms of the basic functions $J_0$, $J_A$, $J_B$, $J_C$, $J_D$, $J_E$, $I_0$, $I_A$, $I_B$, $I_C$, $I_D$, $I_E$ and $K_0$, all of which depend on the momenta $k$ and $p$ :

$$\Lambda^\mu(k,p) \;=\; \sum_{i=1}^{6}\bar{P}_1^i\,V_i^\mu\qquad , \tag{39}$$

where

$$\bar{P}_1^i \;=\; \frac{\alpha}{4}\,P_1^i \tag{40}$$

and the explicit expressions for $P_1^i$ are:

$$\begin{aligned}
P_1^1 &= 2J_A - 2J_C + (\xi-1)\;2p^2 I_D\\
P_1^2 &= 2J_B - 2J_E + (\xi-1)\;2k^2 I_D\\
P_1^3 &= -4J_0 + 4J_A + 4J_B - 2J_D\\
&- \frac{1}{3}(\xi-1)\Big(4J_0 - 6J_A + 2k^2 I_C +\;4k\cdot p\,I_D -\;4p^2 I_E\Big)\\
P_1^4 &= 2J_0 - 2J_A - 2J_B - 2J_D\\
&+ \frac{1}{3}(\xi-1)\Big(2J_0 - 6J_A + 4k^2 I_C -\;4k\cdot p\,I_D -\;2p^2 I_E\Big)\\
P_1^5 &= 3(J_0 - J_A - J_B) + (\xi-1)\,(J_0 - J_A - J_B)\\
P_1^6 &= \frac{1}{3}(\,-6k\cdot p\,J_0 + 3(2k\cdot p\,-k^2)J_A + 3(2k\cdot p\,-p^2)J_B\\
&\qquad +K_0 + 2k^2 J_C + 4k\cdot p\,J_D + 2p^2 J_E)\\
&+ \frac{1}{3}(\xi-1)(\,-2k\cdot p\,J_0 - 3k^2 J_A - 3p^2 J_B\\
&\qquad +3K_0 + 2k^2 k\cdot p\,I_C + 4(k\cdot p)^2 I_D + 2p^2 k\cdot p I_E)\;.
\end{aligned} \tag{41}$$

This is the complete one loop correction to the QED3 vertex in any covariant gauge for massless fermions.

# 3 Analytic Structure of the Vertex

## 3.1 The Longitudinal vertex

**$F(p^2)$ in perturbation theory:**

As explained in Sect. 2.1, owing to the Ward-Takahashi identity, the longitudinal component of the vertex is determined by the fermion function, $F(p^2)$. In perturbation theory to order $\mathcal{O}(\alpha)$, one has to evaluate the graph in Fig. 2. The corresponding mathematical equation is:

$$iS_F^{-1}(p) \;=\; iS_F^{0\;-1}(p) + e^2 \int \frac{d^3k}{(2\pi)^3}\, \gamma^\mu\, S_F^0(k)\, \gamma^\nu\, \Delta^0_{\mu\nu}(q) \qquad , \tag{42}$$

The photon propagator can be split into the transverse and the longitudinal parts as:

$$\Delta^0_{\mu\nu}(q) \;=\; \Delta^{0\;T}_{\mu\nu}(q) - \xi\, \frac{q_\mu q_\nu}{q^4} \qquad , \tag{43}$$

where

$$\Delta^{0\;T}_{\mu\nu}(q) \;=\; -\frac{1}{q^2}\left[g_{\mu\nu} - q_\mu q_\nu/q^2\right] \qquad . \tag{44}$$

Burden and Roberts (see Eq. (25) of [11]) have noted that the solution of Eq. (42) is gauge covariant (in the sense of the Landau-Khalatnikov transformations [12]) if the condition

$$\int \frac{d^3k}{(2\pi)^3}\, \gamma^\mu\, S_F(k)\, \Gamma^\nu(k,p)\, \Delta^{0\;T}_{\mu\nu}(q) \;=\; 0 \tag{45}$$

is simply satisfied. This condition Burden and Tjiang [4] have called the *transversality condition*. It is easy to check that at one loop order this condition is indeed fulfilled and so we are left with

$$iS_F^{-1}(p) \;=\; iS_F^{0\;-1}(p) + e^2 \int \frac{d^3k}{(2\pi)^3}\, \gamma^\mu\, S_F^0(k)\, \gamma^\nu \left(-\xi\, \frac{q_\mu q_\nu}{q^4}\right) \qquad . \tag{46}$$

Substituting the values of $S_F(p)$ and $S_F^0(p)$, then taking the trace after having multiplied with $\not{p}$ and simplifying, we get

$$\frac{1}{F(p^2)} \;=\; 1 - i\xi\, \frac{e^2}{p^2} \int \frac{d^3k}{(2\pi)^3}\, \frac{k\cdot p}{k^2 q^2} \qquad , \tag{47}$$

which can also be written as

$$\frac{1}{F(p^2)} = 1 - i\xi \, \frac{e^2}{2p^2} \int \frac{d^3k}{(2\pi)^3} \, \frac{k^2+p^2-q^2}{k^2q^2} \qquad . \tag{48}$$

Using dimensional regularization, one can see that the last term (and also the first term after appropriate change of variables) is zero as there are no external momenta present in the integrand. Eq. (1) in the appendix simplifies the result to

$$F(p^2) = 1 - \frac{\alpha\xi}{4} \frac{\pi}{\sqrt{-p^2}} + \mathcal{O}(\alpha^2) \tag{49}$$

in the Minkowski space. So the longitudinal vertex to $\mathcal{O}(\alpha)$ is:

$$\Gamma_L^\mu = \left[1 + \frac{\alpha\xi}{4}\sigma_1\right]\gamma^\mu + \frac{\alpha\xi}{4}\sigma_2 \left[k^\mu \not{k} + p^\mu \not{p} + k^\mu \not{p} + p^\mu \not{k}\right] \qquad , \tag{50}$$

where

$$\begin{aligned} \sigma_1 &= \frac{1}{2}\left[\frac{\pi}{\sqrt{-k^2}} + \frac{\pi}{\sqrt{-p^2}}\right] \qquad , \\ \sigma_2 &= \frac{1}{2}\,\frac{1}{(k^2-p^2)}\left[\frac{\pi}{\sqrt{-k^2}} - \frac{\pi}{\sqrt{-p^2}}\right] \qquad . \end{aligned} \tag{51}$$

**Comparison with LKF transformations:**

Assuming that $F(p^2)=1$ in the Landau gauge, LKF transformations yield the following expression for it in an arbitrary gauge:

$$F(p^2) = 1 - \frac{\alpha\xi}{2\sqrt{-p^2}} \tan^{-1}\left[\frac{2\sqrt{-p^2}}{\alpha\xi}\right] \qquad . \tag{52}$$

Using the expansion $\tan^{-1}(1/x) = \pi/2 - x + x^3/3 + \cdots$ for $\mid x \mid << 1$, we get

$$F(p^2) = 1 - \frac{\pi\,\alpha\xi}{4\sqrt{-p^2}} - \frac{\alpha^2\xi^2}{4p^2} + \mathcal{O}(\alpha^3) \tag{53}$$

which is in accordance with the perturbative result to $\mathcal{O}(\alpha)$. Therefore, the LKF transformations accompanied by the assumption that $F(p^2)=1$ in the Landau gauge are in accordance with perturbation theory at the one loop level. A similar comparison at the too loop level is discussed in Sect. 4.

**Remark:**

Burden and Tjiang [4] propose the following non-perturbative expression for $F(p^2)$:

$$F(p^2) = 1 - \frac{\alpha(\xi-\xi_0)}{2\sqrt{-p^2}} \tan^{-1}\left[\frac{2\sqrt{-p^2}}{\alpha(\xi-\xi_0)}\right] \qquad . \tag{54}$$

From the arguments given above, it is easy to see that this expression agrees with the one loop perturbative result only if $\xi_0 = 0$, unlike what is suggested by them.

## 3.2 The Transverse Vertex

Having calculated the vertex to $O(\alpha)$ , Eq. (11,12,39-41), we can subtract from it the longitudinal vertex of Sect. 3.1, Eq. (51,52) and obtain (Eq. (9)) the transverse vertex to $O(\alpha)$. This is given by

$$
\begin{aligned}
\Gamma_T^\mu(k,p) \;=\; \frac{\alpha}{4}\Bigg[\; & k^\mu \not k \quad \{\, 2J_A - 2J_C - \sigma_2 \,+\, (\xi-1)\,(2p^2 I_D - \sigma_2)\,\} \\
& + p^\mu \not p \quad \{\, 2J_B - 2J_E - \sigma_2 \,+\, (\xi-1)\,(2k^2 I_D - \sigma_2)\,\} \\
& + k^\mu \not p \quad \{\, -4J_0 + 4J_A + 4J_B - 2J_D - \sigma_2 \\
& \qquad -\frac{1}{3}\,(\xi-1)\,\Big(4J_0 - 6J_A + 2k^2 I_C +\, 4k\cdot p\, I_D -\, 4p^2 I_E + 3\sigma_2\Big)\,\} \\
& + p^\mu \not k \quad \{\, 2J_0 - 2J_A - 2J_B - 2J_D - \sigma_2 \\
& \qquad +\frac{1}{3}\,(\xi-1)\,\Big(2J_0 - 6J_A + 4k^2 I_C -\, 4k\cdot p\, I_D -\, 2p^2 I_E - 3\sigma_2\Big)\,\} \\
& + \gamma^\mu \not k \not p \, \{\, 3\,(J_0 - J_A - J_B) + (\xi-1)\,\,(J_0 - J_A - J_B)\,\} \\
& + \gamma^\mu \qquad \{\, \frac{1}{3}\,(\,-\,6k\cdot p\, J_0 + 3(2k\cdot p\,-k^2) J_A + 3(2k\cdot p\,-p^2) J_B \\
& \qquad\quad + K_0 + 2k^2 J_C + 4k\cdot p\, J_D + 2p^2 J_E - 3\sigma_1) \\
& \qquad +\frac{1}{3}\,(\xi-1)\,(\,-\,2k\cdot p\, J_0 - 3k^2 J_A - 3p^2 J_B \\
& \qquad\quad + 3K_0 + 2k^2 k\cdot p\, I_C + 4(k\cdot p)^2 I_D + 2p^2 k\cdot p I_E - 3\sigma_1)\,\}\Bigg]
\end{aligned} \tag{55}
$$

in terms of 6 of the vectors $V_i^\mu$. Our task is then to express this result in terms of the 4 basis vectors defining $\Gamma_T^\mu(k,p)$, Eq. (10). Thus from Eq. (9) we can alternatively write out

$$
\begin{aligned}
\Gamma_T^\mu \;=\;& k^\mu \not k \;\; \Big[\tau_2(p^2 - k\cdot p) - \tau_3 + \tau_6\Big] \\
+\;& p^\mu \not p \;\; \Big[\tau_2(k^2 - k\cdot p) - \tau_3 - \tau_6\Big] \\
+\;& k^\mu \not p \;\; \Big[\tau_2(p^2 - k\cdot p) + \tau_3 - \tau_6 + \tau_8\Big] \\
+\;& p^\mu \not k \;\; \Big[\tau_2(k^2 - k\cdot p) + \tau_3 + \tau_6 - \tau_8\Big] \\
+\;& \gamma^\mu \not k \not p \; [-\tau_8] \\
+\;& \gamma^\mu \qquad \Big[\tau_3 q^2 + \tau_6(p^2 - k^2) + \tau_8(k\cdot p)\Big] \qquad .
\end{aligned} \tag{56}
$$

Comparing Eqs. (55) and (56), we have 6 equations for the 4 unknown $\tau_i$. Since $\Gamma_T^\mu$ is transverse to the vector $q_\mu$, Eq. (4), only 4 of these equations are independent. The solution yields expressions for the 4 transverse coefficients $\tau_i$. Each is a function of $k^2, p^2, q^2$ and $\xi$. The results are as follows:

$$
\begin{aligned}
\tau_2 \;=\;& \frac{\alpha\pi}{8\Delta^4} \left\{ \frac{1}{\sqrt{-k^2p^2q^2}}\, q^2 \left[(k\cdot p)^2 + k^2p^2\right] \right. \\
& + \frac{1}{\sqrt{-k^2}}\, \frac{1}{(k^2-p^2)} \left[\Delta^2(p^2 + k\cdot p) - 2k\cdot p\,(k^2-p^2)(k^2 - k\cdot p)\right] \\
& - \frac{1}{\sqrt{-p^2}}\, \frac{1}{(k^2-p^2)} \left[\Delta^2(k^2 + k\cdot p) + 2k\cdot p\,(k^2-p^2)(p^2 - k\cdot p)\right] \\
& \left. + \frac{1}{\sqrt{-q^2}}\, 2q^2 k\cdot p \right\} \\
+\;& \frac{\alpha\pi(\xi-1)}{8\Delta^4} \left\{ - \frac{1}{\sqrt{-k^2p^2q^2}} \left[(k^2+p^2)\Delta^2 + 2k^2p^2q^2\right] \right. \\
& + \frac{1}{\sqrt{-k^2}}\, \frac{1}{(k^2-p^2)} \left[\Delta^2(p^2 + k\cdot p) - 2k^2(k^2-p^2)(p^2 - k\cdot p)\right] \\
& - \frac{1}{\sqrt{-p^2}}\, \frac{1}{(k^2-p^2)} \left[\Delta^2(k^2 + k\cdot p) + 2p^2(k^2-p^2)(k^2 - k\cdot p)\right] \\
& \left. - \frac{1}{\sqrt{-q^2}}\, 2\left[q^2 k\cdot p + \Delta^2\right] \right\} \quad , \qquad (57)
\end{aligned}
$$

$$
\begin{aligned}
\tau_3 \;=\;& \frac{\alpha\pi}{16\Delta^4} \left\{ - \frac{1}{\sqrt{-k^2p^2q^2}} \left[-4(k\cdot p)^2\Delta^2 + (k^2-p^2)^2\left((k\cdot p)^2 + k^2p^2\right)\right] \right. \\
& + \frac{1}{\sqrt{-k^2}} \left[\Delta^2(p^2 - k\cdot p) + 2k\cdot p\,(k^2-p^2)(k^2 + k\cdot p)\right] \\
& + \frac{1}{\sqrt{-p^2}} \left[\Delta^2(k^2 - k\cdot p) - 2k\cdot p\,(k^2-p^2)(p^2 + k\cdot p)\right] \\
& \left. + \frac{1}{\sqrt{-q^2}} \left[2k\cdot p\left(2\Delta^2 - (k^2-p^2)^2\right)\right] \right\} \\
+\;& \frac{\alpha\pi(\xi-1)}{16\Delta^4} \left\{ - \frac{1}{\sqrt{-k^2p^2q^2}} \left[\Delta^2(k^2+p^2)^2 - 2(k\cdot p)^2(k^2-p^2)^2\right] \right. \\
& + \frac{1}{\sqrt{-k^2}} \left[(\Delta^2 + 2k^2p^2)(p^2 + k\cdot p) - 2k^2 k\cdot p(k^2 + k\cdot p)\right] \\
& + \frac{1}{\sqrt{-p^2}} \left[(\Delta^2 + 2k^2p^2)(k^2 + k\cdot p) - 2p^2 k\cdot p(p^2 + k\cdot p)\right] \\
& \left. + \frac{1}{\sqrt{-q^2}} \left[-2k\cdot p\left(2\Delta^2 - (k^2-p^2)^2\right)\right] \right\} \quad , \qquad (58)
\end{aligned}
$$

$$\tau_6 = \frac{\alpha\pi(\xi-2)}{16\Delta^4}\left\{-\frac{1}{\sqrt{-k^2p^2q^2}}\left[(p^2-k^2)q^2\left((k\cdot p)^2+k^2p^2\right)\right]\right.$$
$$-\frac{1}{\sqrt{-k^2}}\left[\Delta^2(p^2-\ k\cdot p\ )+2k^2\left(p^2(p^2-k\cdot p)+k\cdot p(k^2-k\cdot p)\right)\right]$$
$$-\frac{1}{\sqrt{-p^2}}\left[\Delta^2(\ k\cdot p\ -k^2)-2p^2\left(k^2(k^2-k\cdot p)+k\cdot p(p^2-k\cdot p)\right)\right]$$
$$\left.+\frac{1}{\sqrt{-q^2}}\left[2k\cdot p\ q^2(k^2-p^2)\right]\right\} \qquad , \tag{59}$$

$$\tau_8 = \frac{\alpha\pi(\xi+2)}{4\Delta^2}\left\{\frac{-k\cdot p\,q^2}{\sqrt{-k^2p^2q^2}}+\frac{k^2-k\cdot p}{\sqrt{-k^2}}+\frac{p^2-k\cdot p}{\sqrt{-p^2}}-\frac{q^2}{\sqrt{-q^2}}\right\} \qquad . \tag{60}$$

These $\tau_i$ are given in an arbitrary covariant gauge specified by $\xi$, written in the Minkowski space. Any non-perturbative vertex *ansatz* should reproduce Eqs. (57-60) in the weak coupling regime. Therefore, Eqs. (57-60) should serve as a guide to constructing non-perturbative vertex in QED3.

- The $\tau_i$ have the required symmetry under the exchange of vectors $k$ and $p$. $\tau_2$, $\tau_3$ and $\tau_8$ are symmetric, whereas $\tau_6$ is antisymmetric.

- None of the $\tau_i$ has kinematic singularity when $k^2 \to p^2$. Although $\tau_2$ has explicit factors of $(k^2-p^2)$ in the denominator, the terms containing them obviously cancel out in the limit $k^2 \to p^2$.

- All the $\tau_i$ only depend on basic functions of $k$ and $p$. This is unlike the case of QED4 where the $\tau_i$ involve spence functions.

It is instructive to take the asymptotic limit $\mid k^2 \mid >> \mid p^2 \mid$ of the transverse vertex, as another check of the correctness of Eqs. (57-60):

$$\tau_2 \overset{|k^2|>>|p^2|}{=} -\frac{\alpha}{16k^4}\,\frac{\pi}{\sqrt{-p^2}}\,(2-3\xi)\ +\ \mathcal{O}(1/k^5) \tag{61}$$

$$\tau_3 \overset{|k^2|>>|p^2|}{=} -\frac{\alpha}{32k^2}\,\frac{\pi}{\sqrt{-p^2}}\,(2+3\xi)\ +\ \mathcal{O}(1/k^3) \tag{62}$$

$$\tau_6 \overset{|k^2>>|p^2|}{=} -\frac{\alpha}{32k^2}\,\frac{\pi}{\sqrt{-p^2}}\,(2-\xi)\ +\ \mathcal{O}(1/k^3) \tag{63}$$

$$\tau_8 \overset{|k^2>>|p^2|}{=} -\frac{\alpha}{4k^2}\,\frac{\pi}{\sqrt{-p^2}}\,(2+\xi)\;+\;\mathcal{O}(1/k^3)\,. \tag{64}$$

Note that

- the factors of $\Delta^2$ in each of the $\tau_i$ cancel out and there is no dependence on the angle between $k$ and $p$, as expected.
- taking into account the asymptotic limit $\mid k^2 \mid >> \mid p^2 \mid$ of the corresponding basis vectors, one can easily see that $\tau_3$ and $\tau_6$ provide the dominant contribution to $\Gamma_T$ in this limit as in QED4.

Therefore, the complete transverse vertex in the limit $\mid k^2 \mid >> \mid p^2 \mid$ can be written as

$$\Gamma_T^\mu(k,p) \overset{|k^2|>>|p^2|}{=} \frac{\alpha\xi}{8}\,\frac{\pi}{\sqrt{-p^2}}\left[-\gamma^\mu + \frac{k^\mu \not{k}}{k^2}\right] \qquad . \tag{65}$$

This result is strikingly similar to that found in QED4, apart from a factor of $\ln(k^2/p^2)$ replaced with $\pi/\sqrt{-p^2}$. Note that in this limit, the exact QED3 vertex matches onto the proposed Curtis-Pennington vertex [5].

# 4 $F(p^2)$ to Two Loops and Transversality condition

## 4.1 $F(p^2)$ to Two Loops

We have seen that the transversality condition, i.e, Eq. (45), holds true to one loop level. Assuming it to be true non-perturbatively for $\xi = \xi_0$, which we have shown to be equal to zero, Burden *et. al.* [4], have proposed a vertex anstaz, which they then use to solve the photon propagator equation. A crucial test of the validity of their vertex ansatz is checking the transversality condition to two loop order. This is equivalent to calculating $F(p^2)$ to the same level. We carry out this exercise in this section.

The equation for $F(p^2)$ can be extracted from Eq. (42) by multiplying the equation with $\not{p}$ and taking the trace. On Wick rotating to the Euclidean space and simplifying, this equation can be written as:

$$
\begin{aligned}
\frac{1}{F(p^2)} \;&=\; 1 \;-\; \frac{\alpha}{2\pi^2 p^2} \int \frac{d^3k}{k^2} \frac{F(k^2)}{q^2} \\
&\quad \Bigg[ a(k^2,p^2)\, \frac{2}{q^2} \left\{ (k\cdot p)^2 - (k^2+p^2) k\cdot p + k^2 p^2 \right\} \\
&\quad + b(k^2,p^2) \left\{ (k^2+p^2) k\cdot p + 2k^2p^2 - \frac{1}{q^2}(k^2-p^2)^2 k\cdot p \right\} \\
&\quad - \frac{\xi}{F(p^2)} \frac{1}{q^2} \left\{ p^2 (k^2 - k\cdot p) \right\} \\
&\quad + \tau_2(k,p) \left\{ -(k^2+p^2)\Delta^2 \right\} \\
&\quad + \tau_3(k,p)\, 2 \left\{ -(k\cdot p)^2 + (k^2+p^2) k\cdot p - k^2p^2 \right\} \\
&\quad - \tau_6(k,p)\, 2 \left\{ (k^2-p^2) k\cdot p \right\} \\
&\quad + \tau_8(k,p) \left\{ \Delta^2 \right\} \Bigg] \qquad , \qquad (66)
\end{aligned}
$$

where

$$
a(k^2,p^2) = \frac{1}{2}\left( \frac{1}{F(k^2)} + \frac{1}{F(p^2)} \right), \qquad b(k^2,p^2) = \frac{1}{2}\frac{1}{k^2-p^2}\left( \frac{1}{F(k^2)} - \frac{1}{F(p^2)} \right) \; . \qquad (67)
$$

In connection with carrying out the integral in the above equation, it is convenient to write $\tau_i$, Eq. (57-60), in the Euclidean space, and adopt the notation $k=\sqrt{k^2}$, $p=\sqrt{p^2}$, $q=\sqrt{q^2}$. The only angular dependence is hence displayed in $q=\sqrt{k^2+p^2-2kp\mathrm{cos}\theta}$:

$$
\tau_2 \;=\; \frac{\alpha\pi}{4} \frac{1}{kp(k+p)(k+p+q)^2} \left[ 1 + (\xi-1)\, \frac{2k+2p+q}{q} \right] \, , \qquad (68)
$$

$$
\tau_3 \;=\; \frac{\alpha\pi}{8} \frac{1}{kpq(k+p+q)^2} \left[ 4kp + 3kq + 3pq + 2q^2 + (\xi-1)\,(2k^2+2p^2+kq+pq) \right] , \qquad (69)
$$

$$
\tau_6 \;=\; \frac{\alpha\pi(2-\xi)}{8} \frac{k-p}{kp(k+p+q)^2} \, , \qquad (70)
$$

$$
\tau_8 \;=\; \frac{\alpha\pi(2+\xi)}{2} \frac{1}{kp(k+p+q)} \, . \qquad (71)
$$

We substitue Eqs. (68-71) in Eq. (66). Again employing the standard technique to identify $2k\cdot p = (k^2+p^2-q^2)$ and making use of $d^3k = 2\pi dk k^2 d\theta \mathrm{sin}\theta$, we carry out the angular integration:

$$
\begin{aligned}
\frac{1}{F(p^2)} &= 1+\frac{\pi\xi}{4p}\,\alpha-\frac{\alpha^2}{4p^2}\int_0^\infty dk\,\frac{1}{2kp(k+p)} \\
&\left[\frac{\xi}{2}\left(k^2-p^2\right)\left\{-(k^2-p^2)^2 I_4+I_0\right\}\right. \\
&+\frac{\xi}{2}\left\{(k^2-p^2)^2(k^2+p^2)I_4-2(k^2+p^2)^2 I_2+(k^2+p^2)I_0\right\} \\
&-\xi^2 p^2(k^2-p^2)\left\{(k^2-p^2)I_4+I_2\right\} \\
&+\{(k+p)\left(2kp(k-p)^2 I_3+(k-p)^2(k+p)I_2-(k^2+p^2)I_1-(k+p)I_0+I_{-1}\right) \\
&\quad+\xi((k-p)^2(2k^2+2p^2+3kp)I_2-(k+p)(k^2+p^2-4kp)I_1 \\
&\quad\left.-(2k^2+2p^2+3kp)I_0+(k+p)I_{-1})\}\right] \quad , \qquad (72)
\end{aligned}
$$

where

- the first curly-bracket expression arises from the $a$–term in Eq. (65), the second one from the $b$–term, the third from the $\xi/F(p^2)$-term and the fourth from the transverse part of the vertex. On substituting $I_4$, $a$–term vanishes identically as it does at one loop level. Note that all the $(k+p+q)$ factors in the $\tau_i$ neatly cancel out, leaving us with simpler integrals to be evaluated.

- and the $I_n$ are defined as

$$
I_n = \int_0^\pi d\theta\,\frac{\sin\theta}{q^n}
$$

with the evaluated expressions given in the appendix.

Keeping in mind the form of the integrals $I_n$, we divide the integration region in two parts, $0 \rightarrow p$ and $p \rightarrow \infty$. For the first region, we make the change of variables $k = px$ and for the second region, $k = p/x$. On simplification, we arrive at

$$
\begin{aligned}
\frac{1}{F(p^2)} &= 1+\frac{\pi\xi}{4p}\alpha+\frac{\alpha^2\xi^2}{8p^2}\int_0^1\frac{dx}{x}\,\left[2-(1-x)^2\mathcal{L}\right] \\
&-\frac{\alpha^2}{24p^2}\int_0^1\frac{dx}{x^2}\left[-4x^2(x+1)-6(3x^2+1)+3(1-x^2)^2\mathcal{L}\right] \\
&-\frac{\alpha^2\xi}{8p^2}\int_0^1\frac{dx}{x^2}\left[-\frac{2}{3}(2x-1)(x^2-3x-3)+(1+x)^2(x^2-3x+1)\mathcal{L}\right] , \qquad (73)
\end{aligned}
$$

where

$$
\mathcal{L} = \frac{1}{x}\ln\frac{1+x}{1-x} \quad . \qquad (74)
$$

The above integrals can be evaluated in a straight forward way. In order to make a direct comparison with Eq. (53), we prefer to write the final expression in Minkowski space by substituting $p \rightarrow \sqrt{-p^2}$ and $p^2 \rightarrow -p^2$:

$$F(p^2) \;=\; 1 - \frac{\pi\,\alpha\xi}{4\sqrt{-p^2}} - \frac{\alpha^2\xi^2}{4p^2} + \frac{3\alpha^2}{4p^2}\left(1+\frac{\pi^2}{12}\right) - \frac{\alpha^2\xi}{2p^2}\left(1-\frac{\pi^2}{4}\right) + \mathcal{O}(\alpha^3)\,. \qquad (75)$$

One can note various important features of this result:

- $F(p^2) \neq 1$ in the Landau gauge.
- The existence of constant and $\mathcal{O}(\xi)$ terms at $\mathcal{O}(\alpha^2)$ implies the violation of the transversality condition. We shall elaborate more on this remark in Sect. 4.2.
- Eq. (52) is derived from the LKF transformations based upon the assupmtion that $F = 1$ in the Landau gauge. As we have seen, this assumption is not correct to $\mathcal{O}(\alpha^2)$, and therefore, Eq. (52) is not expected to hold true in general, as is confirmed on comparing Eq. (53) and Eq. (75). However, a comparison between the two results suggests that it contains the correct $\mathcal{O}(\xi^2)$ term at the level $\mathcal{O}(\alpha^2)$, though it does not reproduce other terms appearing in the exact perturbative calculation.

## 4.2 Burden and Tjiang Transversality Condition

The perturbative expression for $F(p^2)$ to the two loops shows that the Burden-Tjiang transversality condition does not hold true beyond one loop order. Now we explicitly calculate the left hand side of Eq. (45). In the most general form, it can be expanded as:

$$i\int \frac{d^3k}{(2\pi)^3}\,\gamma^\mu\, S_F(k)\,\Gamma^\nu(k,p)\,\Delta^{0\;\,T}_{\mu\nu}(q) \;=\; A(p^2) + B(p^2)\,\not{p} \quad , \qquad (76)$$

where the multiplication with $i$ is only for mathematical convenience. $A(p^2)$ and $B(p^2)$ can be extracted by taking the trace of the above equation, having multiplied by 1 and $\not{p}$ respectively. With the bare fermion being massless, it is easy to see that on doing the trace algebra and contracting the indices, $A(p^2) = 0$. Our evaluation of $F(p^2)$ helps us identify $B(p^2)$ from Eq. (75) so that:

$$i\int \frac{d^3k}{(2\pi)^3}\,\gamma^\mu\, S_F(k)\,\Gamma^\nu(k,p)\,\Delta^{0\;\,T}_{\mu\nu}(q) =$$
$$\left[-\frac{3\alpha}{16\pi p^2}\left(1+\frac{\pi^2}{12}\right) + \frac{\alpha\xi}{8\pi p^2}\left(1-\frac{\pi^2}{4}\right) + \mathcal{O}(\alpha^2)\right]\;\not{p}\,. \qquad (77)$$

Obviously, for $\xi = 0$,

$$i \int \frac{d^3k}{(2\pi)^3}\, \gamma^\mu\, S_F(k)\, \Gamma^\nu(k,p)\, \Delta^{0\ \ T}_{\mu\nu}(q)\ |_{\xi=0} \ = \ \left[ -\frac{3\alpha}{16\pi p^2}\left(1+\frac{\pi^2}{12}\right) + \mathcal{O}(\alpha^2)\right] \not{p}\ , \quad (78)$$

which is a violation of the transversality condition at the two loop level.

# 5 Conclusions

In this paper, we present the one loop calculation of the fermion-boson vertex in QED3 in an arbitrary covariant gauge for massless fermions. In the most general form, the vertex can be written in terms of 12 independent Lorentz vectors. Following the procedure outlined by Ball and Chiu, 4 of the 12 vectors define the longitudinal vertex. It satisfies the Ward-Takahashi identity which relates it to the fermion propagator. The transverse vertex is written in terms of the remaining 8 vectors. For massless fermions, only 4 of these vectors contribute. Subtraction of the longitudinal vertex from the full vertex yields the transverse vertex. We evaluate the coefficients of the basis vectors for the transverse vectors to $\mathcal{O}(\alpha)$. Moreover, using this result, we calculate $F(p^2)$ analytically to $\mathcal{O}(\alpha^2)$ and find that the transversality condition does not hold true to this order. Therefore, any non-perturbative construction of the transverse vertex based upon this condition cannot be correct.

Knowing the vertex in any covariant gauge may give us an understanding of how the essential gauge dependence of the vertex demanded by its Landau-Khalatnikov transformation [11, 12] is satisfied non-perturbatively. Moreover, the perturbative knowledge of the coefficients of the transverse vectors provides a reference for the non-perturbative construction of the vertex as every *ansatz* should reduce to this perturbative result in the weak coupling regime. The evaluation of $F(p^2)$ to $\mathcal{O}(\alpha^2)$ in an arbitrary covariant gauge should also serve as a useful tool in the hunt for the non-perturbative vertex which is connected to the former through Ward-Takahashi Identity and the Schwinger-Dyson equations. Any vertex ansatz must reproduce Eq. (75) for $F(p^2)$ to $\mathcal{O}(\alpha^2)$ when the coupling is weak, leading to a more reliable non-perturbative truncation of Schwinger-Dyson equations.

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# Appendix

Following are the integrals used in the calculation presented in the paper:

$$\begin{aligned} Q_1 &= K^{(0)} = \int_M d^3w \, \frac{1}{(k-w)^2 \, (p-w)^2} \\ &= \frac{i\pi^3}{\sqrt{-q^2}} \end{aligned} \tag{1}$$

$$\begin{aligned} Q_2^\mu &= \int_M d^3w \, \frac{w^\mu}{(k-w)^2 \, (p-w)^2} \\ &= \frac{i\pi^3}{2\sqrt{-q^2}} \, (k+p)^\mu \end{aligned} \tag{2}$$

$$\begin{aligned} Q_3 &= \int_M d^3w \, \frac{1}{w^4 \, (k-w)^2} \\ &= 0 \end{aligned} \tag{3}$$

$$\begin{aligned} Q_4 &= I^{(0)} = \int_M d^3w \, \frac{1}{w^4 \, (k-w)^2 \, (p-w)^2} \\ &= \frac{k \cdot p}{k^2 p^2} \, J^{(0)} \end{aligned} \tag{4}$$

$$Q_5 = J^{(0)} = \int_M d^3w \, \frac{1}{w^2 \, (p-w)^2 \, (k-w)^2}$$

$$J_0 = \frac{2}{i\pi^3} \, J^{(0)} \; = \; \frac{-2\pi}{\sqrt{-k^2 p^2 q^2}} \tag{5}$$

$$I_{-1} = \frac{2}{3kp} \, \Big[ p(3k^2+p^2)\theta(k-p) + k(k^2+3p^2)\theta(p-k) \Big] \tag{6}$$

$$I_0 = 2 \tag{7}$$

$$I_1 = \left[ \frac{2}{k}\theta(k-p) + \frac{2}{p}\theta(p-k) \right] \tag{8}$$

$$I_2 = \frac{1}{2kp} \, \ln \frac{(k+p)^2}{(k-p)^2} \tag{9}$$

$$I_3 = \frac{2}{kp(k^2-p^2)} \, [p\theta(k-p) - k\theta(p-k)] \tag{10}$$

$$I_4 = \frac{2}{(k+p)^2(k-p)^2} \tag{11}$$

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## Figures

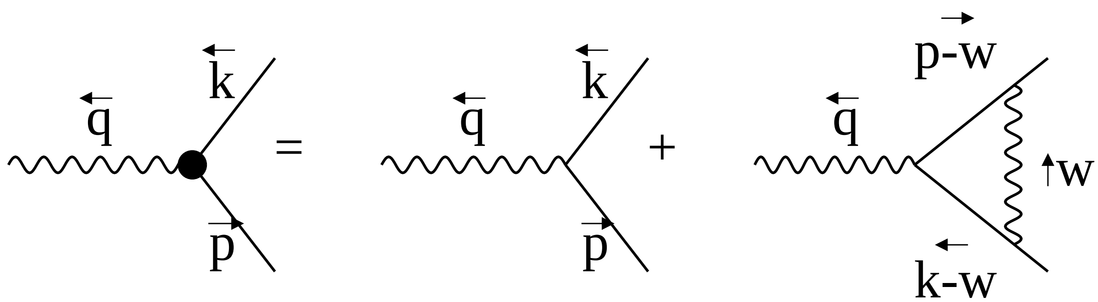


**Fig. 1.** One loop correction to the vertex.

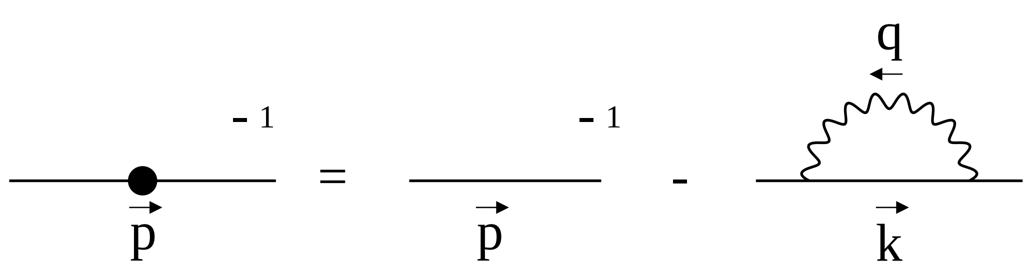


**Fig. 2.** One loop correction to the fermion propagator.